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Trade matrices for trade negotiators, applied to 'Brexit' and 'NAFTA': an approach

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Abstract

This paper puts forth a new scholarly approach to trade negotiations, for practitioners of international agreements, or simply to business students attempting to understand Ricardian trade theory. The paper hypothesizes that matrices can provide a simpler conceptual framework for considering Ricardo's comparative advantage, especially when multiple goods and multiple countries are involved, in order to determine which countries should produce which goods. Numerous theoretical examples are presented, singularly, and jointly, as are different possible flaws and assumptions, additional applications, and alternative uses of the matrices, such as employing matrices to increase production of certain goods needed during crises or shortages. The article also argues that "terms of trade" should not be "assumed" in trade models but be based upon indifference curves, and addresses other influencing factors such as neoclassical changes in utility or in production. Found valid, the paper applies this method of trade simplification to pressing international situations, the question of "Brexit," the sobriquet for the United Kingdom's effort to withdraw from the European Union, which creates interesting possibilities for new trade deals, and the renegotiation of the North American Free Trade Agreement (NAFTA). The conclusion conceptually compares bilateral and multilateral trade, singularly, and with all countries together.

Keywords: matrices; trade; comparative advantage; Ricardo; bilateralism

JEL Classification: A22, F11, F13, M16.

1. Intro, Research Question, and Hypothesis

Ricardian economics, the study of specialization in trade, could use a simpler method of understanding for students and practitioners. Matrices have long been used in economics for determining optimal outcomes, beginning with the works of the French economist Leon Walras in the late 1800s, who related the prices of a multiple-good economy to each other, using a so-called *numeraire*, or estimate for the very first, basic good. Thus, he created a so-called "general equilibrium"

of all price relationships in an economy (Walras, 1874). In March, 2018, United States Trade Representative Robert Lighthizer testified before Congress that he had, in fact, been studying matrices, of some sort, to help him address the North American Free Trade Agreement (NAFTA). He seeks higher percentages of parts made domestically for automobiles, greater working standards, and renegotiating NAFTA every 5 years, as opposed to the Canadians' desire for deal making every 10 years. The former seems useful but cumbersome (The Committee 2018). Trade of numerous goods in an economy is very complex, as is Ricardian economics. Are there better ways to understand complex trade deals, both scholarly, and for real-life practice as well, for simplification purposes? This article hypothesizes that matrices, either used via paper or with computers, and by summing their production numbers from the diagonals of such diagrams, can make international trade easier to understand for practitioners, and students, alike.

Theory:

Early European economists, such as Hume and Smith, theorized that trade brings more goods to developing countries (Pressman 2014). Perhaps the value of goods traded has changed, but in effect, most trade today brings more products from low cost countries to wealthier countries, who trade fewer of their own wealthier products. Adam Smith believed in an absolute theory of production, that countries should produce and trade the good that they make most efficiently, irrelevant of other countries. Brilliantly, however, the theory for "comparative advantage" came just later from David Ricardo, who showed that trade is relative. Essentially, countries specialize in goods they are *relatively* productive in, and then the countries trade, overall increasing the number of goods that are produced and obtained, and therefore, maximizing utility, or happiness. Ricardo believed in a supply based production system, relying on Say's Law, that supply determines demand. However, the "terms of trade," or ratio of one good traded for another, need to be nailed down based on demand and utility, rather than based on supply assumptions. Sometimes, a value-based system can yield greater utility to certain countries able to take advantage of "terms of trade," but not necessarily more utility for all. Still, this phenomenon is sometimes obtainable in simpler trade "systems" or pacts of countries trading with each other, briefly shown later.

2. Literature Review

Matrices have a span of uses ranging from simple organization to breaking data into categories or years, such as most recently in Godlewska-Majkowska and Komor (2017), who studied automobile production. Matrices are more commonly used to solve values for multiple variables when enough equations exist. Or, they can be used to weigh societal choices, through "decision-matrices," such as in

Campoy-Munoz, P. et al. (2017). More famously, a matrix can help determine business, or political-economic, strategies, through game theory choice outcomes. Regarding trade, Le Fouler et al. (2001) use matrices, at a time when the United States was in recession, to trace how slow growth across developed countries led to low export growth, export performance, and import and export prices. Najarzadeh et al. (2011) ranked characteristics of goods in a matrix, such as revenues and profits, to help determine Iranian export competitiveness. Most importantly, textbooks, such as those by Salvatore (1996) and Grieco and Ikenberry (2003), which deal with trade, as well as most articles on trade, discuss two countries with two goods, on an x and y axis, or, presented in a simple 2x2 box "matrix," which they do not even call it as such. But, these authors do in fact mention that the same format would hold true for multiple goods, using three, or multiple, dimensional axis such as a "z axis" along the lines of multivariable calculus. Still, such graphs can imaginably become extremely complex with numerous graphs, such that matrices can be used for simpler understanding.

3. Method and Results

This article creates simple matrices for trade to determine if they can indicate optimized production, in a simpler way, through diagonal summation. In Grieco and Ikenberry's example of a 2x2 matrix, it is apparent that the diagonal at which the most number of goods produced is greatest, or the reciprocal, that the opportunity cost of the good is least, is the ideal situation, maximizing output. That holds true in matrices of 3x3 goods and countries, in which there are 9 possible matrices and 18 possible diagonals (see Appendix A), or in a 4x4 model, in which 64 possible diagonals would be possible (4^(4-1)). Some of the diagonals which are the similar backwards and forward can be eliminated. Presumably, this concept of using the diagonal in a matrix should hold for even higher number of goods as well. Trade negotiators could use computers to create thousands of matrices, and have the computer identify which diagonal produces the most goods, or at the lowest cost. Observe the original example below:

Table 1: A Two Country-Two Good Trade Model

	Country A	**Country B**
	good a: 4	good a: 10
	good b: 6	good b: 3

Use of the diagonal maximizes production. In production terms, 10 + 6 is clearly greater than 4 + 3, or any vertical or horizontal summation. And, if using opportunity costs, or simply the costs of production, which would be the reciprocal, the numbers would be lower, and the conclusion the same. This diagram then indicates which countries should focus on producing which goods. If there happened to be ties in terms of diagonals, then utility, or happiness, would come into effect, since one country would value trading more of one good for its own product. In the figure below, we see how David Hume may have been correct in that small countries may benefit more greatly, comparatively, than larger ones.

Table 2: Large- vs. Small-Production Trade

	Country A	**Country B**
	good a: 0	good a: 2
	good b: 7	good b: 2

Even though both countries benefit, the less productive country benefits more by receiving more goods through trade, proportionately (goods received vs. total at end), assuming it trades 1:1 for good b, or better, though the larger nation gains variety. In this case, comparative advantage, which is based on relative cost, might be called a "competitive advantage," which consists of other advantages in companies or countries in reaching consumers within a market. Comparative advantage holds, as many might not realize, only if one of the country's goods is equal to or double half the value of the average of another country's two goods: "quasi-advantage." Otherwise, absolute advantage, which Adam Smith believed in, would take hold, because one country would simply divide its output. Averages are used not only since they maximizes production, but because production utilities are assumed to be the same, as are "terms of trade," to be discussed. Partially finished goods are assumed to be eventually completed. This paper now considers more complex trade, with three countries. Complex matrices will be explained in greater detail later on in this paper, after discussing three-way trade, in deals like "Brexit" and "NAFTA," which are ongoing at the time of this writing.

Table 3: A Three Country-Three Goods Diagonal Trade Model

Country A	Country B	Country C
good a	good a	good a
good b	good b	good b
good c	good c	good c

(one of 9 possible matrices and 18 diagonals: see Appendix)

Matrices can be used to determine all possibilities for production, such as in the three-by-three example above, and again, one diagonal, or several tied, of the matrices would yield the greatest production. Simply taking the largest productions of each good would not adhere to opportunity costs, or goods foregone in production, the concept on which relative, "comparative advantage," is based.

Table 4: A Four Country-Four Goods Diagonal Trade Model

Country A	Country B	Country C	Country D
a	a	a	a
b	b	b	b
c	c	c	c
d	d	d	d

Caption: Complex decisions are simplified by the diagonal.

Besides intercountry analysis, orthodox examination assumes production within countries (intra-country) are the same, but the possibility also exists that different segments of very diverse countries, such as in the provinces of India, can produce goods at different rates. Krugman (1981) showed that this occurs within large, multinational corporations. And, if in producing a very inexpensive product, that product has great value, then it may make sense for countries not to produce based on comparative advantage, but comparative value. If countries' consumers value a good more highly than another in a two-good model, this could cause a country to want to produce or trade far more of it, but its consumption would, in most cases, be solely of that product, unless the other good were a staple, when trade would be desirable. This presents interesting future research. Nevertheless, this is a truism that negotiators should be aware of. Observe the model below:

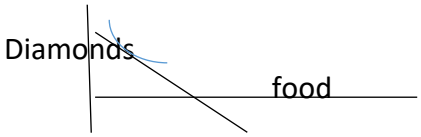

Figure 1: A Luxury Versus Necessity Production Figure

Caption: Utility can result in desire for specific goods, such as luxury goods.

If countries, or consumers, value goods completely differently from one country to another, which is a key assumption, then trade becomes irrelevant, as shown above, which is why cultural goods, such as snow shovels in Canada, may not spread, as described in Barry (2015). As another example, "Buy American" movements, or such movements in other countries favoring nationalistic goods, may cause consumers to value domestic goods more. And, if prices are higher in one country for a good than another, trade will be of a fewer number of goods, because of the higher cost, but utility will be equal or higher, since quantity x price = value, or Gross Domestic Product (GDP). If countries value goods differently from each other, utility maximization can be possible without production maximization, but it is unlikely that each country would benefit individually in terms of utility, thus making trade unlikely. In cases where diagonals have ties in production, then utility maximization should be preeminent by using indifference curve relationships and prices, discussed below. "Offer curves," which depict how many of one good countries will trade for another country's goods, which are featured prominently in textbooks, should be based on utility relationships in indifference curves, and not the reverse, which is where "terms of trade" applies.

4. Additional Analysis/Assumptions

Trade not only depends on output, but also on demand and utility. Utility is best accounted for in so-called *"terms of trade,"* the ratio at which one country trades a product for another country's goods, which typically is left to assumption in most text books, and should ideally not be. In calculating the utility in trade, one should start with a domestic level of utilities. Demand between countries may be very different, but this is accounted for in part by utility in indifference curves, upon which consumers are indifferent between goods. Indifferent curves, when tangent to production curves, indicate the relationship between goods that the consumers of a country prefer, and can obtain, via production or trade. A terms of trade advantage, from producing even more wealthier goods, or increased buying power by larger nations, could lead to greater gains from trade; terms of trade, with a three good model, would be: a/b, b/c, a/c, for each country trading

with every other one. Terms of trade for the same good would most likely not be all the same for the same countries, since they value goods differently- if so, such as under a trade pact, then they most likely are, and should, be based on averages. One must assume that terms of trade are the same for all countries for simplification, if the countries are operating in a trade organization or "system" that ensure "fairness," but this assumption will be revisited later on.

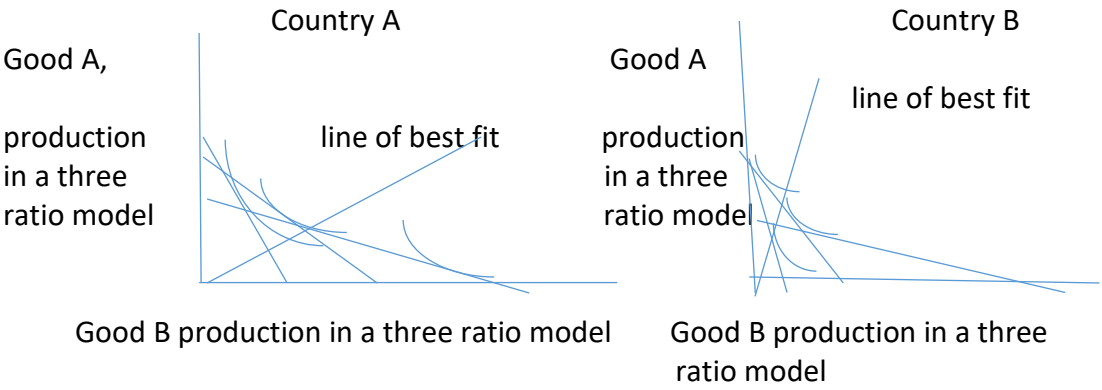

Figure 2: A One-Dimensional Utility Line of Best Fit Using Changeable Goods as Axes

The figure above is meant to show the "utility line of best fit." In most text books, prices are used to determine the "terms of trade," how much of one good is traded for another, whether in terms of prices, or in terms of barter. Prices are often assumed. Prices should be used if available, but if not, then using a line of best fit is better than using raw assumptions. Utility or prices as determined domestically result in the outcome after trade which most closely reflects the domestic country's preferences, given by the line of best fit, which travels directly between the points where the indifference curves meet the production curves. It is a supply oriented model, per Ricardo: countries consider what they give up, first, rather than what they gain, contrary to how individuals typically think of trade, or any transaction. In other words, one good on an indifference curve is compared to another, which is then compared to the line of best fit, and then the other country. This should maximize domestic utility since utility from one good is assumed to be akin to another. Utility on its own cannot be completely compared between two goods in such cases, philosophically, other than noting that one is further from the origin point.

In their simple model, for "terms of trade," Grieco and Ikenberry (2003) "assume" that 10 million U.S. computers are traded for 100 million Brazilian shoes, but rather than using assumptions, the relationship between the goods traded should come as close as possible to the relationships within each country of utility

between goods. This relationship is expressed along the indifference curves, and this method would satisfy the original utility relationships. Since the utility in one country cannot be said to be more important than in another, then taking an average of the output relationship of the two goods in production of the 8/3 (2.66) U.S. relationship, and the 175/3 (23.33) Brazilian relationship, or 13 [(2.66 + 23.33)/2]. But, overall utility increases most, and is closest to the line of best fit, with a ratio of the two prices, thus 8.75 (23.33/2.66).

This terms of trade value must fall within the maximum ranges of production for countries and goods, or else trade would either be impossible or unnecessary. Any amount of trade, determined by demand, in the heretofore ratio should satisfy the initial consumption preferences. If the utility relationships within each country are not known, then policy-makers can consider using prices as a proxy for utility. Grieco and Ikenberry's assumption of a 10:1 relationship is not far off, but it benefits, in terms of the utility relationship, the United States slightly over Brazil in their example (10 is closer to 2.66 than 23.33).

Terms of trade in some models are determined by offer curves, but the curves must be assumed to be based on utility preferences from indifference curves, and terms of trade can be influenced by: larger countries having stronger buying power, by exchange rates, which depend on supply and demand for goods and/or assets that affect the relationship of prices, by changing preferences that affect demand, or changes in resources affecting supply. Tariffs can strengthen terms of trade because they increase prices, but may result in utility-losing trade wars through higher prices to consumers. If larger countries purchase in bulk, this can lower foreign prices and thus consumption costs, lessening the terms of trade.

5. Discussion

In determining whether a country could be valuable to a trade pact, a country's productivity of one good is often compared to the average productivity of the countries all together, a so-called index (Salvatore 1996). This method is the easiest and most convenient way to analyze trade in a complex, multi-good and -country situation, although a further in-depth application follows this discussion section. Simply from conceptualizing this case, we can see that adding a country with low productivity for a new good could lessen the overall average productivity and result in less total output, unless the production of all of the countries where to reorient themselves towards better combined comparative advantage. This lessening is unlikely, however, even if countries adhere to producing only what they are best at, but neoclassical economics shows how factors between countries are constantly changing, from the movement of capital and, sometimes, labor, causing productivities to change over time. This activity

gives support for newer trade deals to be renegotiated periodically. Additionally, adding countries to trade deals might increase and spur competition to producing and trading wealthier products.

As neoclassical theory (Grieco and Ikenberry 2003) shows, as did Barry (2017) in detail, a country may observe a coming advantage in a trade pact if it sees its productivity of a good approaching the productivity of another country's comparatively advantaged good. This would change the relationship between the products, and the trade deals themselves. Utilities for some goods may be increasing, although aggregate utility cannot truly be measured other than if one schedule of products is determined to be at a higher level than another. In the models here, utility has been assigned differently to goods based on indifference curves, but such values are difficult to calculate in real life, other than using prices. But, prices, or prices compared to costs, are nevertheless not always representative. Consider again the example of the "diamond-water paradox," where a valuable good such as water is priced lowly compared to less valuable diamonds. Such difficulty in measuring utility is precisely the "impossibility theorem" offered by economist Kenneth Arrow (Pressman 2014). Moreover, utilities can change from substitution effects, that is, changes in preferences, or by income effects, that is, changes in income or prices.

Overall, the matrices reveal that with equal goods and countries, diagonal calculation is the best way to increase production and efficiency. In addition, the matrices can provide insight into the quantity of goods produced of each good, and the benefit to that particular industry, therefore being an additional advantage of matrices. Albeit, this added information from matrices may be questioned by those who argue against governments "picking winners and losers," skewing government policies towards certain goods if it wishes to. However, such actions might be very important, for instance, if some goods, such as pharmaceuticals, may be needed in case of pandemics, agricultural goods in times of drought, or oil in times of shortages, for example, though focusing on these goods may cause overall output to be less.

6. Application

United States President Donald J. Trump has expressed his opposition to complex trade deals, instead supporting smaller, less complex agreements that obviate the problems heretofore mentioned. Complex trade deals could be problematic if countries do not make the same number of goods, if data for productivity within countries is not available, or if data is calculated differently for or by different countries. But, rough production costs can be used as appropriate

proxies. The main drawback to large international trade deals in this setting is if countries have similar productivities to other countries, or if countries have similar productivities within their countries for different goods. In these cases, they would not trade, since it would not make sense to, unless the trade is "intra-industry," whereby similar goods may have slight differences to cater to specific cultural tastes and preferences. In addition, production diagram curves should technically slope downwards, because productivity, or factors, are sacrificed in switching from one good to another. In light of these perspectives, as productivities converge globally, all future trade may become "intra," which Krugman (1981) pointed out. Another drawback to complex agreements is the lack of diversification amongst goods, which can increase risk, especially in terms of shocks or other crises, for which matrices can be helpful to prevent.

Despite all of this information, how can this method be applied to real life? First, assume that the production levels are according to the schematic below, with no tariffs. Two out of the three goods, a and c, are produced *not* at their highest level, showing that complex trade can distort production.

Table 5: Assumption Model for Three-Country Scenario

	Country A	Country B	Country C
Good a: ipads	60	190	49
Good b: shoes	20	180	31
Good c: cars	48	140	45

President Trump has expressed the possibility of greater bilateral trade deals. The United States and the United Kingdom are currently negotiating the "Transatlantic Trade and Investment Partnership," or T-TIP, with Europe, that was set back by revelations by U.S.-intelligence-employee Edward Snowden that the United States was presumably spying on Germany. Such a deal seems initially to be more economically oriented than political, as opposed to the Trans-Pacific Partnership, and research shows it has the potential to create 13 million new jobs in the United States and European Union (United States Trade Representative 2018b). However, the United States' desire for a narrower U.S.-U.K. trade deal actually predates to the presidency of George H.W. Bush, the progenitor of many of the deals signed after his administration, including the North American Free Trade Agreement (NAFTA). The United Kingdom's ostensible leaving of the European Union, currently led by Prime Minister Theresa May, called 'Brexit,' will leave a void of vast imports, which the United States could potentially fill.

Of British trade, 47% of the United Kingdom's exports are to the European Union (EU), while 51% of its imports are from the European Union, a larger percentage of imports. The U.K. relies on the EU for trade, mostly in wholesale and retail trade, transport, and food services, a vast gap left open possibly for the United States, which currently supplies 9% of British imports (Europa 2018). The United States is still the seventh largest exporter to the U.K., and the United Kingdom has $480 billion dollars invested in the United States, while the United State has $600 billion dollars invested in the U.K. (U.S. Chamber of Commerce 2018). The United Kingdom's exports which the United States might be able to pick up could be aircraft (where U.K. to EU exports are $31.1 billion yearly), some machinery where productivity is rising ($29.9 billion), and certain intra-industry trade in optical and medical instruments ($25.4 billion). It seems unlikely that the United States would import mineral fuel or energy products. The United States also might be able to import some agricultural products, though the U.S. tastes do not match up exactly with British production, and also, the United States might benefit from some of London's financial services (United States Trade Representative, 2018b). Services amounted to 39% of the U.K.'s 2017 exports to the EU: 56% of these were in two categories, those being financial services, and "other business services," which involves "legal, accounting, advertising, research and development, architectural, engineering [, etc.] services" (Ward 2018, 3).

Conversely, the United States could help the United Kingdom with supply chains, particularly for shipping and otherwise transporting goods or intermediate parts, with limits on goods that do not contain a certain percentage of complete manufacturing in the host country. The United States could also ship the United Kingdom machinery (where exports to the EU are $64.9 billion annually), pharmaceuticals ($55.2 billion annually), vehicles ($54.6 billion annually) and possibly chemicals, wine and beer, or the agricultural goods needed to produce them, which have high tariffs for imports in the United Kingdom (United States Trade Representative" 2018a). A two country matrix with numerous goods may lack the total number of diagonals mentioned earlier, but a counterfactual country could be added for additional goods. The counterfactual could be one of the two countries themselves, determined by which combination is greatest, but now with 32 unique diagonals, which are observed in the very basic model below, not including those diagonals eliminated for being similar to each other.

Table 6: An Uneven Diagonal Trade Model

U.S.	U.K.	Either
a	a	a
b	b	b
c	c	c

Let us now consider this matrix with a real life example whereby the United Kingdom produces (good "a") administrative support services with a productivity index of 112.7, and the United States does with a productivity index of 106, where for good "b," internet and communication services, the United Kingdom has a productivity index of 98.6, and the United States 111.7, and where for good "c," finance and insurance services, the United Kingdom has a productivity index of 95, and the United States has one for 98.6. The data is for 2015 from the Organisation for Economic Cooperation and Development, for gross value added per hour worked for employed persons. This problem is a straight forward example, except for the fact that two goods within the two countries both have a productivity of 98.6.

Table 7: A Possible U.S.-U.K. Production Model

	U.K.	U.S.	***U.S.***/U.K.
Good a: admin.	112.7	106.3	106.3
Good c: fin.	95	98.6	98.6
Good b: comm.	98.6	111.7	111.7

The example above only satisfies maximum conditions if, assuming that a country can only produce one good, or else other production will need to be halved, assuming production time does not change. In other words, the third good added, here assumed to be communications, must already exist or be established in the economy, or else production of this good along with the countries other good would be halved, or 1/3 if producing three goods. However, the good could still be added if, by combining with another good, it is equal to or greater than the halved average of another country's two goods (Country B and goods b/c and a/c and Country A and b/c and a/c). If two goods are already "established," adding a third country with a third good could completely distort comparative advantages, as shown below in the three-country trade versus two:

Table 8: Changed Specialization

Country A	Country B	Country C
Good a: 10	Good a: 1	Good a: 9
Good b: 3	Good a: 15	Good b: 8
Good c: 7	Good c: 1	Good c: 5

For a two goods, two country model, when adding a third country, the first two countries may choose to divide production of a good between the countries to maximize output. If production of a third good is added to this particular model, then the entire matrix must be revised, but adding the third country assumes that it would add to production. For a three country, four good model, the non-established good of the fourth country would have to be equal or greater than the averages of two or three goods of a country, noted later. For a three country, two good model, any value of a third good is acceptable, but if another country develops a good C which is higher in production, specialization of the other goods may change. And, the added good must be equal or higher than the halved values of the same country's two goods. Otherwise, two-way trade suffices. These remarks will be reapplied later.

In the above example, there are two optimal diagonals amidst the 18 (3x3x2) matrices, some of the matrices which are eliminated for similarity, but both show the same result. The table shows which country should produce which good based on the most productive diagonal using index data. The simple matrix indicates that the United Kingdom would in fact be able to benefit the United States by exporting good a, administrative support services, while the United States should produce and export good c, finance and insurance services, and the United States also produce and export good b, Internet and communication services. The exportation of services is becoming more standard, especially due to globalization, and was a main component of the Trans-Pacific Partnership (TTP). Currently the United States and the world are moving slowly away from free trade "at all costs," because of President Donald J. Trump's partiality for trade deals which are reciprocal from the standpoint of the countries involved.

For NAFTA, the United States would tend to produce greater high-tech goods like computers, whereas more labor abundant countries would be able to better produce foods and beverages, and resource rich countries, wood products. From this simple table below, using 2015 output values from the OECD, it is apparent that the system which maximizes total output centers on the diagonal below, with the blue lines indicating all possible trade patterns.

Table 9: NAFTA Renegotiation Example Model

	U.S.	Mexico	Canada
Good a: computers	139 bill.	877 mill.	1.03 bill.
Good b: beverages	5.84 bill.	1.84 bill.	144 mill.
Good c: wood goods	195 mill.	59.5 mill.	84 mill.

This model makes apparent that in separate trade deals, countries would be able to place individual tariffs on different countries (each arrow of trade would be treated separately), assuming that multi-nation trade deals do not already. Also, terms of trade could be altered by nations being able to buy or sell at different prices to individual countries rather than setting them constant to all. Separate policies might be provisions of some trade deals, but behaving otherwise might be deemed an infraction by the World Trade Organization (WTO). Exact calculations of this case will be left to future research.

Comparing a dual country matrix with a three country matrix as President Trump desires with NAFTA, production may theoretically be less, but utility may be closer to the original utility relationship. Moving away from utility relationships change the economy more via tastes and preferences, and leads to greater changing and movement of factors like investment that affect productivity over time, though overall utility should be greater, due to wealth effects. Assuming that countries produce different goods, the variety of products increase, due to the trade of a new good, but the indifference curves will move farther away from their original positions.

For NAFTA, changing from a two good, two country model to a three good, three country model, it must be assumed that the original production of the third good is zero with the original countries, such that any addition of a third country with an additional good is beneficial. Still, adding a country and a good can lower overall average productivity. In addition, adding a fourth country to a three country, four good model, and again assuming that countries start by producing one good at a time, or else production is halved, then production of the fourth good would have to be equal to or greater than the average of two of the other goods for each country, or one-third if one country already produces two goods, and so forth. In spite of this, in the models that follow, dissecting a three country back to a two country model assumes that the countries can in fact produce the third good. Again, with two countries, and three goods, if the third good's production is two times greater than the average of a country's two other two goods, then the countries will benefit from a third country added. Or, if not for this good, the countries benefit if any added good is twice the value of the average of another states' two goods, one of which is the added good, or, it would not make sense to add the good, and another country should produce it. In almost all cases adding a good would increase production, except that adding a good may drastically change the comparative advantages such that countries should produce different goods. If new goods are tied productively with the average, the trade is fruitless, unless it is intra-industry, serving specific tastes. Average is

assumed because it keeps the production relationship between the two original goods the same, cutting them each in half for half of the time of production.

Moving from three countries with three goods, and reducing to two goods traded three ways, which President Trump favors with NAFTA, can completely change the comparative advantages. The possibility exists that overall utility may slightly increase (Table 9) from bilateralism! Or, countries' terms of trade may change in two ways, as opposed to three ways, trade, in this simple example, which assumes trade for goods is done individually, and without tariffs (Tables 11, 12, and 13). Such change may cause a country to have to choose whether to attempt to produce more, or to gain more goods through trade, the last case being the more probable and beneficial. Finally, one country's production specialty can change from one good to another (Table 14). In this last conceptual example, removing a third country from trade, as suggested by President Trump with NAFTA, not only increases utility, but obviates the need for the third state; Country A gains a new specialty, as does country B, while Country C is left out entirely. Country C must have stayed in the three-way trade due to a bad deal originally created, or from changing productivities over time which distorted the original comparative advantage relationships.

Table 10: Changing From Three Countries to Two

Country A	Country B	Country C
Good A: 20	Good A: 1	Good A: 2
Good B: 3	Good B: 18	Good B: 4
Good C: 7	Good C: 6	Good C: 5

A	B
Good A: 20	Good A: 1
Good B: 3	Good B: 18

A	B
Good A: 20	Good A:1
Good C: 7	Good C: 6

B	C
Good C: 6	Good C: 5
Good B: 18	Good B: 4

With Table 10 above, the 2 way trades should be halved, not shown for conceptuality, but they split because countries are now focusing on two goods rather than one, and have less time for production than focusing on one good. But, in this scenario, whereas with three-way trade, Country A was the dominant trader with the highest production, at 20, here, Country B comes out as the big winner in terms of number of goods traded, but, it must be careful to make sure it conserves enough for the domestic populace. In additional, overall utility has increased by 0.5. This increase is from increased flexibility and options of trade, by B being able to trade 6. This increased utility would not have occurred had B's production of this good had been less. Thus, bilateral trade deals can sometimes be preferable to multilateral trade. Table 11 below shows typical a/b, a/c, b/c, trade, without a change in specialization, but how terms of trade *may* change since lines of best fit have shifted; this would alter the number of each good traded. This is assuming that all trade takes place in this example as individual countries sending goods individually, as opposed to in groups, an assumption pertinent for future research, both theoretically and empirically. This model also assumes that there are no tariffs, which could also alter individual goods' terms of trade.

Table 11: Utility Relationships: Trade Combined

Country A	Country B	Country C
Good A: 20	Good A: 1	Good A: 2
Good B: 3	Good B: 18	Good B: 4
Good C: 7	Good C: 2	Good C: 5

Table 12: Calculations of utility relationships: each good/average of all of the summed country's goods

Country A ratios 3 ways,	Country B ratios 3 ways,	Country C ratio 3 ways
good A: 20/(30/3)= 2,	1/(21/3)= 0.143	2/(11/3)= 0.545
good B: 3/(30/3)= 0.3,	18 (21/2)= 2.57	4/(11/3)= 1.09
good C: 7/(30/3)= 0.7,	2/ (21/3)= 0.286	5/(11/3)= 1.36

Table 13: Individual, Separate Trade

A	B	A/B ratios 2 ways
Good A: 20	Good A: 1	Country A, good A ratio: 20/(23/2)= 1.74, lessened
Good B: 3	Good B: 18	Country B, good B ratio: 18/(19/2)= 1.89, lessened

A	C	
Good A: 20	Good A:1	Country A, good A ratio: 20/(27/2)= 1.48, lessened

Good C: 7	Good C: 5	Country C, good C ratio: 5/(6/2)= 1.67, increased

B C

Good C: 6	Good C: 5	Country C, good C ratio: 5/(9/2)= 1.11, lessened
Good B: 18	Good B: 4	Country B, good B ratio: 18/(24/2)= 1.5, lessened

Table 14: Changing Specializations

Country A	Country B	Country C
Good A: 10	Good A: 14	Good A: 6
Good B: 3	Good B: 15	Good B: 6
Good C: 9	Good C: 16	Good C: 8

A B

Good A: 10	Good A: 1
Good C: 7	Good C: 16

A B

Good A: 10	Good A: 1
Good B: 3	Good B: 15

A B

Good B: 3	Good B: 15
Good C: 9	Good C: 16

The main advantage of trading with countries individually would be to affect the terms of trade separately, though this might be considered illegal under a complex, World Trade Organization (WTO) pact. Adding a certain good may drastically change terms of trade for another good, using the best fit line. Also, time is needed to raise the productivity of a new good to be traded. However, a country taking time and resources to produce that new good may lower production time for, and productivity of, the goods currently traded. If terms of trade change by more than the relationship of the goods change, one country may gain more than another. In some cases, it may not be useful for one country to be

part of the trade "system" of tariffs, prices, and terms of trade. Smaller agreements, additionally, are more easily enforceable. And, if a country has more valuable terms of trade for one good with one country, it may decide to produce more of that good, and/or sell more of it to the country valuing it more, with value affecting trade because comparatively advantaged goods are usually tariffed less.

7. Conclusion

In summation, matrices here have been found to be a very helpful tool to trade negotiators operating within international agreements, or, they can simply be a teaching tool for students, depending on how complex to make them. A goal of future research should be to examine even larger sets of numbers, and to determine how to use computers with such theories. This article has also clarified and readdressed some of the assumptions used in calculating the maximum production of goods, and utility, and also noted how the use of the matrices may be used to favor certain goods in times of crises or great need. Matrices could prove a valuable tool in ongoing negotiations with future trade deals, such as "Brexit" or "NAFTA" at the time of this writing, by helping to identify which goods should be stressed by which negotiators, and also where tariffs can be reduced to reduce costs for countries, their so-called "terms of trade," given knowledge of production. And, at times, bilateral trade might be preferable to multilateralism. Ricardian economics is thus simpler, but can lead to greater analysis, by such a tool, depending on its diverse utility.

Appendix A.

Possible Matrices for Three Country-Three Good Trade Model

	A	B	C		A	B	C		A	B	C
a				a				c			
b				c				a			
c				b				b			

	A	C	B		A	C	B		A	C	B
a				a				c			
b				c				a			
c				b				b			

	B	A	C		B	A	C		B	A	C
a				a				c			
b				c				a			
c				b				b			

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